

# AN EFFECTIVE LAGRANGIAN APPROACH TO NUCLEI

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## **Abstract:**

The construction of a general effective lagrangian consistent with the symmetries of QCD and intended for applications to finite-density systems is discussed. The low-energy structure of the composite nucleon is described within the theory from vector-meson dominance. Results are given for finite nuclei and nuclear matter at one-loop order. All of the coupling constants, appropriately defined according to naive dimensional analysis, are found to be natural.

## **Outline:**

- Introduction
- Physical Ingredients
- The Lagrangian
- EM Structure of the Nucleon
- One-Loop Results
- Summary and Future Work

# INTRODUCTION

- Why adopt the approach of an effective hadronic lagrangian?
  - Modern renormalization theory makes sense of nonrenormalizable effective lagrangians.
  - QCD at low energies is equivalent to some effective theories with low-energy degrees of freedom.
  - QHD mean-field theory is successful for nuclei; it is thus interesting to study nuclei from the modern viewpoint of effective theories.
  - The composite structure of the nucleon can be described in increasing detail by adding additional nonrenormalizable interactions.
- Why include non-Goldstone bosons?
  - The  $E^4$  low-energy constants of ChPT are saturated by vector-meson contributions.
  - The important intermediate-range NN interactions are efficiently described by their exchange ( $\sigma$ ,  $\omega$ ,  $\dots$ ).
  - Large  $N_c$  QCD reduces to a nonlinear effective theory of mesons with the nucleon emerging as a soliton.
  - At finite density, nucleon bilinears such as  $\overline{N}N$  and  $N^\dagger N$  develop expectation values that are conveniently and efficiently described by the appropriate meson mean fields.

## PHYSICAL INGREDIENTS

- Chiral Symmetry: Use CCWZ's nonlinear realization of  $SU(2)_L \otimes SU(2)_R$  symmetry.
- Gauge Invariance: Allow for general gauge invariant terms.
- Broken Scale Invariance: A light scalar *might* be associated with the broken scale invariance of QCD.
- NDA and Naturalness: Naive dimensional analysis (NDA) and a “naturalness” assumption justify a truncation of the lagrangian. They are useful as long as the density is not too high.
- Field Redefinitions: Use to simplify the interaction terms between the nucleons and the non-Goldstone bosons.
- Vacuum Effects: Vacuum baryon and non-Goldstone boson loops are “integrated out,” with their effects buried in the remaining coefficients. This disentangles vacuum dynamics from valence nucleon dynamics.

## • Chiral Symmetry

– Pions:

$$U(x) \equiv \xi(x)\xi(x) \ , \quad \xi(x) = e^{i\pi/f_\pi} \ , \quad \pi = \boldsymbol{\tau} \cdot \boldsymbol{\pi}/2$$

– Nucleons:

$$N(x) = \begin{pmatrix} p(x) \\ n(x) \end{pmatrix}$$

– Rho:

$$\rho_\mu = \boldsymbol{\tau} \cdot \boldsymbol{\rho}/2$$

– CCWZ's nonlinear realization of  $SU(2)_L \otimes SU(2)_R$ :

$$(\xi, \rho_\mu, N) \xrightarrow{L \otimes R} (\xi', \rho'_\mu, N')$$

where  $L \in SU(2)_L$ ,  $R \in SU(2)_R$ , and

$$\begin{aligned} \xi'(x) &= L\xi(x)h^\dagger(x) = h(x)\xi(x)R^\dagger \\ \rho'_\mu(x) &= h(x)\rho_\mu(x)h^\dagger(x) \\ N'(x) &= h(x)N(x) \end{aligned}$$

Note

$$h(x) = h(\boldsymbol{\pi}(x), L, R) \in SU(2)_V$$

which also guarantees  $\pi$  to be a pseudoscalar under parity:  $\xi \xrightarrow{P} \xi^\dagger$

## • Gauge Invariance

- Under  $U(1)_{\text{EM}}$ , the fields transform similarly with

$$L = R = h(x) = \exp[i\alpha(x)(\tau_3 + Y)/2]$$

$Y = 0$  for  $\pi$  or  $\rho$  and  $Y = 1$  for  $N$ .

- Allow for the most general gauge invariant terms instead of the usual minimal substitutions.
- As a result, universality of the couplings need **not** be enforced:

$$g \neq g_{\rho\pi\pi} \neq g_\rho \neq g_\gamma$$

where  $g_\rho$  is the  $\rho NN$  coupling and  $g_\gamma$  the rho-photon coupling (see below).

## • Broken Scale Invariance

- A light scalar *might* be associated with the broken scale invariance with its potential constrained by the low-energy theorems. [ Phys. Rev. C52 (1995)1368 ]
- Field redefinitions (see below) make the constraints weak. Therefore we adopt a general potential for the scalar  $\phi$ :

$$V_S(\phi) = m_s^2 \phi^2 \left( \frac{1}{2} + \frac{\kappa_3}{3!} \frac{g_s \phi}{M} + \frac{\kappa_4}{4!} \frac{g_s^2 \phi^2}{M^2} + \dots \right)$$

## • NDA and Naturalness

- NDA rules for assigning a dimensional coefficient of the right size to any term [Georgi], with  $M =$  nucleon mass taken to be the generic cutoff scale:
  1. include a factor of  $1/f_\pi$  for each strongly interacting field, e.g.  $\phi$  or  $\omega_\mu$ ;
  2. include an overall factor of  $f_\pi^2 M^2$ ;
  3. add factors of  $M$  to get the dimension to 4.
- Implied “naturalness” assumption: dimensionless coefficients, after extracting the dimensional factors and some counting factors, are of  $O(1)$ .
- Assuming NDA, a truncation of the lagrangian is effective only if the density is not too high. The reason is that, for nuclear matter at saturation or at the center of finite nuclei, the scalar ( $\phi$ ) and vector ( $\omega$ ) mean fields

$$g_s \phi_0 / M, \quad g_\omega V_0 / M \approx 0.3 \sim 0.4$$

and the gradients on the surfaces of finite nuclei

$$g_s |\nabla \phi_0| / M^2, \quad g_\omega |\nabla V_0| / M^2 \approx (0.2)^2.$$

- Based on this mean-field estimate, we will count a field and a derivative to be of the same order.
- Fits to the properties of nuclei should determine if nature likes the naturalness assumption and the expansion in powers of the fields and their derivatives.

## • Field Redefinitions

- Field redefinitions leave the on-shell  $S$ -matrix invariant and, among other things, they are usually used to remove redundant terms so that there are no terms with  $\partial^2$  acting on a boson field or  $\partial_\mu$  acting on a fermion field *except* in the kinetic terms.
- In finite-density applications, the exchanged non-Goldstone bosons are far off-shell. They are introduced to simulate the intermediate-range interactions; the interaction ranges remain the same since the field redefinitions do not change the quadratic terms.
- Higher-order derivatives on the non-Goldstone fields are small compared to their masses and need not be eliminated.
- We use the field redefinitions to simplify the interaction terms between the nucleon and the non-Goldstone bosons such that they are essentially of the Yukawa form. For example, the following terms are redundant:

$$\overline{N}N\phi^2, \overline{N}N\phi^3, \overline{N}N\partial^2\phi, \overline{N}N\omega_\mu\omega^\mu, \dots$$

- It is still convenient to remove all the derivatives on the nucleon fields except in the kinetic energy term.

## • Vacuum Effects

- The QCD vacuum is modified by interactions with valence nucleons. However, the vacuum modifications are not adequately treated in a hadronic model, so they must be disentangled from valence dynamics and the effects incorporated into the coefficients of the effective lagrangian.
- Vacuum baryon and non-Goldstone boson loops are short-distance physics and should be integrated out so that an expansion in powers of the fields and derivatives is possible. But their fields are still needed in the lagrangian to account for the valence nucleons and to treat the mean fields conveniently.
- Formally, these loops can be eliminated by introducing counterterms, which is not a problem since *all possible interaction terms are already there*.

## THE LAGRANGIAN

Split the lagrangian into the nucleon and meson parts:

$$\mathcal{L} = \mathcal{L}_N + \mathcal{L}_M.$$

The nucleon part is

$$\begin{aligned} \mathcal{L}_N(x) = & \bar{N}(i\gamma^\mu \mathcal{D}_\mu + g_A \gamma^\mu \gamma_5 a_\mu - M + g_S \phi)N \\ & - \frac{f_\rho g_\rho}{4M} \bar{N} R_{\mu\nu} \sigma^{\mu\nu} N - \frac{f_\omega g_\omega}{4M} \bar{N} \omega_{\mu\nu} \sigma^{\mu\nu} N \\ & - \frac{e}{4M} F_{\mu\nu} \bar{N} \lambda \sigma^{\mu\nu} N \\ & - \frac{e}{2M^2} \bar{N} \gamma_\mu (c_S + c_V \tau_3) N \partial_\nu F^{\mu\nu} + \dots \end{aligned}$$

where  $\sigma_{\mu\nu} = i[\gamma_\mu, \gamma_\nu]/2$ , the nucleon covariant derivative is



$$\mathcal{D}_\mu = \partial_\mu + iv_\mu + ig_\rho \rho_\mu + ig_\omega \omega_\mu + \frac{i}{2}eA_\mu(1 + \tau_3)$$

with the vector and axial vector fields defined as

$$v_\mu = -\frac{i}{2}(\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger), \quad a_\mu = -\frac{i}{2}(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger),$$

the field strength tensors are, with  $R_\mu \equiv \rho_\mu + v_\mu/g$ ,

$$\begin{aligned} \omega_{\mu\nu} &= \partial_\mu \omega_\nu - \partial_\nu \omega_\mu, & F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu \\ R_{\mu\nu} &= \partial_\mu R_\nu - \partial_\nu R_\mu + ig[\rho_\mu, \rho_\nu] \end{aligned}$$

and the magnetic-moment operator of the nucleon

$$\lambda = \frac{1}{2}\lambda_p(1 + \tau_3) + \frac{1}{2}\lambda_n(1 - \tau_3),$$

with  $\lambda_p = 1.79$  and  $\lambda_n = -1.91$  the anomalous moments.

- The  $\omega$  meson can be considered as a singlet in  $SU(2)$  symmetry.
- Terms of next order are

$$\bar{N}\rho_{\mu\nu}\sigma^{\mu\nu}N\phi, \quad \bar{N}\omega_{\mu\nu}\sigma^{\mu\nu}N\phi, \quad \dots$$

which are small since the tensor terms are already small and the main effects of the above are slight modifications of  $f_\rho$  and  $f_\omega$  at finite density due to the nonvanishing scalar expectation value.

- Nucleon contact terms such as

$$\bar{N}N\bar{N}N, \quad \bar{N}\gamma_\mu N\bar{N}\gamma^\mu N, \quad \dots$$

have been taken into account by fitting  $g_s$  and  $g_\omega$ . In other words, the dominate effects of other particles in the same scalar and vector channels are implicit in  $g_s$  and  $g_\omega$ .

Up to quartic order, the meson part of  $\mathcal{L}$  is

$$\begin{aligned}
\mathcal{L}_M(x) = & -\frac{1}{2}\text{tr}(R_{\mu\nu}R^{\mu\nu}) - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi \\
& -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{f_\pi^2}{4}\text{tr}(\partial_\mu U\partial^\mu U^\dagger) \\
& - 2f_\pi^2 e A^\mu \text{tr}(v_\mu \tau_3) - \frac{eF_{\mu\nu}}{2g_\gamma} [\text{tr}(\tau_3 R^{\mu\nu}) + \frac{1}{3}\omega^{\mu\nu}] \\
& + \frac{1}{2}\left(1 + \eta_1 \frac{g_s\phi}{M} + \frac{\eta_2 g_s^2\phi^2}{2M^2}\right)m_\omega^2\omega_\mu\omega^\mu \\
& + \frac{1}{4!}\zeta g_\omega^2(\omega_\mu\omega^\mu)^2 + \left(1 + \eta_\rho \frac{g_s\phi}{M}\right)m_\rho^2\text{tr}(\rho_\mu\rho^\mu) \\
& - m_s^2\phi^2\left(\frac{1}{2} + \frac{\kappa_3}{3!}\frac{g_s\phi}{M} + \frac{\kappa_4}{4!}\frac{g_s^2\phi^2}{M^2}\right) + \dots
\end{aligned}$$

- Apart from some conventional definitions, the couplings have been defined so that they should be of order unity if the NDA is valid.
- Some unimportant terms, such as those with two photon and two rhos or higher powers of rhos, have been omitted.
- The numerical importance of higher-order terms is checked by including in  $\mathcal{L}$  these fifth-order terms:

$$\begin{aligned}
\mathcal{L}_5 = & \frac{1}{4}\frac{g_s\phi}{M}[2\alpha_1(\partial_\mu\phi)^2 - \alpha_2\omega_{\mu\nu}^2] - \frac{\kappa_5}{5!}\frac{g_s^3\phi^3}{M^3}m_s^2\phi^2 \\
& + \frac{\eta_3}{3!}\frac{g_s^3\phi^3}{M^3} \cdot \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \frac{\zeta_1}{4!}\frac{g_s\phi}{M}g_\omega^2(\omega_\mu\omega^\mu)^2
\end{aligned}$$

# EM STRUCTURE OF THE NUCLEON

The electromagnetic current is a sum of point-nucleon and meson-cloud contributions (pion terms omitted):

$$\begin{aligned}
 J_\mu &= J_\mu^{\text{PT}} + J_\mu^{\text{M}} \\
 J_\mu^{\text{PT}} &= \frac{1}{2} \bar{N} (1 + \tau_3) \gamma_\mu N + \frac{1}{2M} \partial_\nu (\bar{N} \lambda \sigma^{\mu\nu} N) \\
 &\quad - \frac{1}{2M^2} \partial^2 [\bar{N} (c_s + c_v \tau_3) N] \\
 J_\mu^{\text{M}} &= \frac{1}{g_\gamma} (\partial_\nu \rho_3^{\mu\nu} + \frac{1}{3} \partial_\nu \omega^{\mu\nu}) + 2f_\pi^2 \text{tr} (v_\mu \tau_3)
 \end{aligned}$$

from which the EM isoscalar and isovector Dirac and Pauli form factors of the nucleon can be read off:

$$\begin{aligned}
 F_1^{\text{S}}(Q^2) &= \frac{1}{2} - \frac{c_s}{2} \frac{Q^2}{M^2} - \frac{g_\omega}{3g_\gamma} \frac{Q^2}{Q^2 + m_\omega^2} + \dots \\
 F_1^{\text{V}}(Q^2) &= \frac{1}{2} - \frac{c_v}{2} \frac{Q^2}{M^2} - \frac{g_\rho}{2g_\gamma} \frac{Q^2}{Q^2 + m_\rho^2} + \dots \\
 F_2^{\text{S}}(Q^2) &= \frac{\lambda_p + \lambda_n}{2} - \frac{f_\omega g_\omega}{3g_\gamma} \frac{Q^2}{Q^2 + m_\omega^2} + \dots \\
 F_2^{\text{V}}(Q^2) &= \frac{\lambda_p - \lambda_n}{2} - \frac{f_\rho g_\rho}{2g_\gamma} \frac{Q^2}{Q^2 + m_\rho^2} + \dots
 \end{aligned}$$

- The photon **does** see an extended nucleon; there is no need for external “intrinsic” form factors.
- The charge density of a nucleus,  $\rho_{\text{ch}} = \rho^{\text{PT}} + \rho^{\text{M}}$ , has contributions from the neutrons as well as from the meson clouds which, as seen below, smear out the point nucleon charge oscillations and increase the RMS radius.

## ONE-LOOP RESULTS

- A systematic calculation to quartic order requires at least the inclusion of two-loop contributions.
- Calculate only through one loop in this first investigation.
- Parameters are optimized to fit the following observables in  $^{16}\text{O}$ ,  $^{88}\text{Sr}$ , and  $^{208}\text{Pb}$  (using a  $\chi^2$  minimization):

binding energies per nucleon  $E_B/A$ ,  
rms charge radii  $\langle r^2 \rangle^{\frac{1}{2}}$ ,  
spin-orbit splittings  $\Delta E_{\text{SO}}$  of least-bound  
protons and neutrons,  
diffraction minimum sharp radii  $R_{\text{dms}}$ ;  
also the  $1h_{9/2}$  energy level of  $^{208}\text{Pb}$ .

Here  $R_{\text{dms}} \equiv 4.493/Q_0^{(1)}$ , with  $Q_0^{(1)}$  the first zero of the charge form factor of the nucleus in momentum space. The  $1h_{9/2}$  level is used to fix the overall scale of the  $^{208}\text{Pb}$  energy levels, which is strongly affected by  $g_\rho$ .

- Parameters:

Input Parameters (mass in MeV and radius in fm)								
$M$	$m_\omega$	$m_\rho$	$\lambda_p$	$\lambda_n$	$g_\gamma$	$\langle r^2 \rangle_{s1}^{\frac{1}{2}}$	$\langle r^2 \rangle_{v1}^{\frac{1}{2}}$	$\langle r^2 \rangle_{v2}^{\frac{1}{2}}$
939	782	770	1.79	-1.91	5.01	0.79	0.79	0.88

$g_\gamma$  is determined from  $\Gamma_{\rho^0 \rightarrow e^+e^-} = 6.8 \text{ keV}$

Fitted Parameters							
$g_s/4\pi$	$g_\omega/4\pi$	$g_\rho/4\pi$	$\eta_1$	$\eta_2$	$\zeta$	$\kappa_3$	$\kappa_4$
0.78	0.90	0.55	-0.71	1.39	1.20	-0.33	1.79
$m_s/M$	$\eta_\rho$	$f_\omega/4$	$\eta_3$	$\kappa_5$	$\zeta_1$	$\alpha_1$	$\alpha_2$
0.58	-1.11	0.024	-0.24	2.98	0.52	0.49	0.002

According to NDA, all entries are natural if  $O(1)$ . Also we have

$$c_s = \frac{M^2}{6} \langle r^2 \rangle_{s1} - \frac{2}{3} \frac{g_\omega}{g_\gamma} \frac{M^2}{m_\omega^2} = 0.19$$

$$c_v = \frac{M^2}{6} \langle r^2 \rangle_{v1} - \frac{g_\rho}{g_\gamma} \frac{M^2}{m_\rho^2} = 0.31$$

$$\frac{f_\rho}{4} = \frac{\lambda_p - \lambda_n}{24} \frac{g_\gamma}{g_\rho} m_\rho^2 \langle r^2 \rangle_{v2} = 1.33$$

- Results (including center-of-mass corrections)

	$E_B/A$ (MeV)			$\langle r^2 \rangle_{\text{chg}}^{\frac{1}{2}}$ (fm)			$R_{\text{dms}}$ (fm)	
	PC*	here	expt	PC	here	expt	here	expt
$^{16}\text{O}$	7.97	7.98	7.98	2.73	2.70	2.74	2.78	2.76
$^{88}\text{Sr}$	8.75	8.71	8.73	4.21	4.18	4.20	4.98	4.99
$^{208}\text{Pb}$	7.87	7.89	7.87	5.51	5.49	5.50	6.79	6.78

\* Point-coupling model of Nikolaus et al.

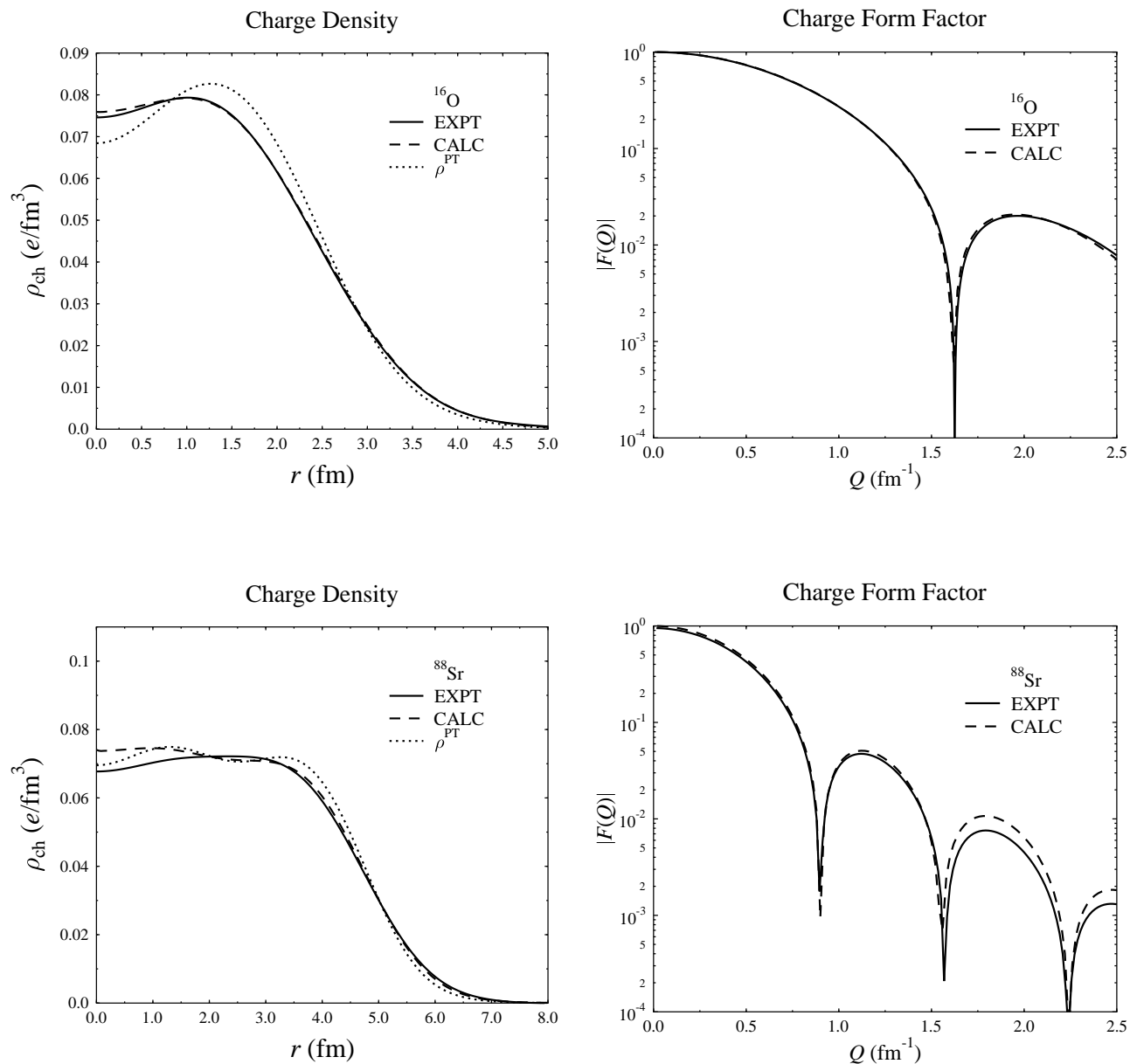
	$\Delta E_{\text{SO}}^{(n)}$ (MeV)			$\Delta E_{\text{SO}}^{(p)}$ (MeV)		
	PC	here	expt	PC	here	expt
$^{16}\text{O}$	6.4	5.8	6.2	6.4	5.8	6.3
$^{88}\text{Sr}$	1.9	2.0	(1.5)**	6.1	5.7	(3.5)**
$^{208}\text{Pb}$	0.91	0.77	0.90	1.96	1.58	1.33

\*\* calculated from Bohr and Mottelson:

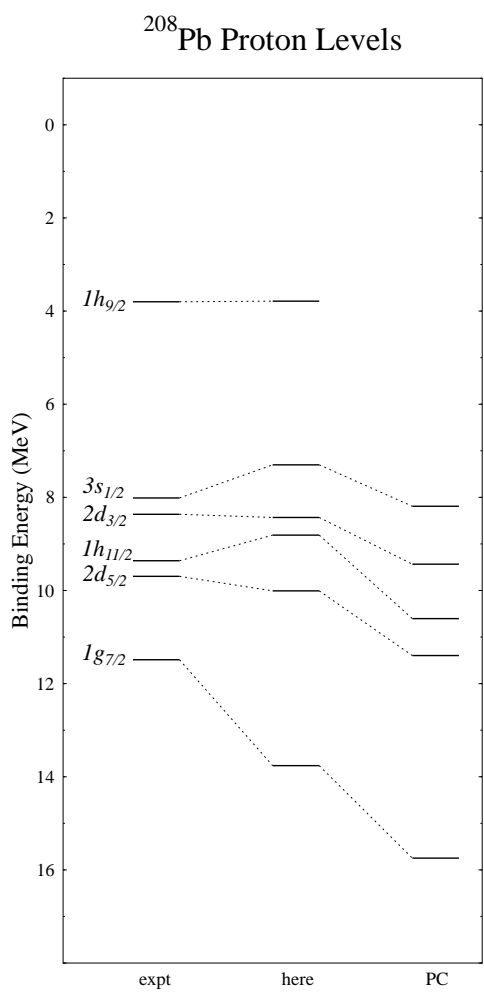
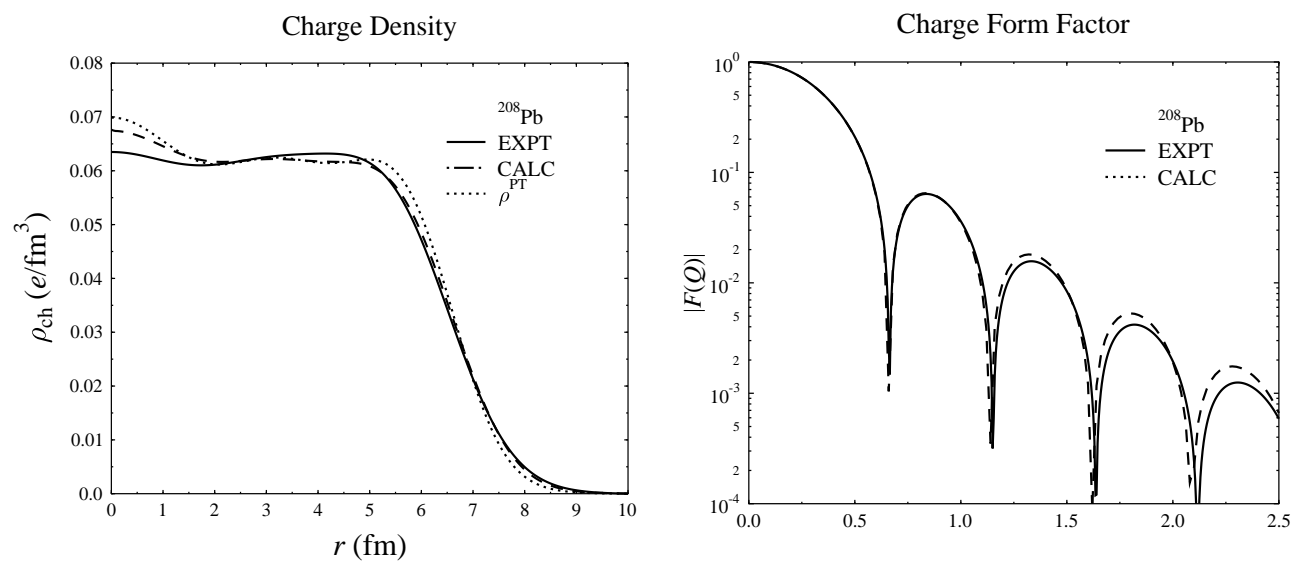
$$\Delta E_{\text{SO}}^{(n)} = \Delta E_{\text{SO}}^{(p)} \approx 10(2l + 1)/A^{2/3} \text{ MeV}$$

– The **predicted** saturation properties of nuclear matter with  $k_F$  the Fermi momentum in  $\text{fm}^{-1}$ ,  $K$  the compressibility,  $a_4$  the symmetry energy, and  $M^*$  effective mass. Also given is the fifth-order contribution ( $\Delta E_5$ ) to  $E_B/A$ . The mass unit is MeV.

$E_B/A$	$k_F$	$K$	$a_4$	$M^*/M$	$g_\omega V_0$	$\Delta E_5$
(MeV)	( $\text{fm}^{-1}$ )	(MeV)	(MeV)		(MeV)	(MeV)
16.1	1.31	214.	37.5	0.622	284	0.45



Note the agreement between the calculated and experimental results at low momenta.





## SUMMARY AND FUTURE WORK

- The low-energy structure of the nucleon and finite nuclei can be well-described by a chiral effective theory of hadrons at one-loop order.
- The NDA and naturalness assumption are indeed compatible with the observed properties of nuclei. The fit parameters are natural and the results are not driven by the last terms kept.
- Descriptions of the spin-orbit splittings of the least bound nucleons and the shell structure are generally good, although there are some discrepancies in the fine details of the energy levels of heavy nuclei.
- The meson clouds play an important role in smearing out the point nucleon charge fluctuations.
- Future work:
  - Two-loop calculations (in progress);
  - Connections with point-coupling lagrangian;
  - Role of the explicit  $\Delta$  degrees of freedom.

*[Note added: A journal article with greater detail and references will be available soon. ]*